Wladis, BMCC/CUNY Name_____

Math 301 Test 2—Take Home Test

You may use the book, notes, web resources, etc. for this exam and you may discuss the exam questions with others, but all of your answers should be your own work, and you should understand your answers well enough that if I asked you to explain any of your answers one-on-one, you would be comfortable doing so.

1. Suppose there are two mystery operations, the *arrow* and the *three dots*, for example: $\vec{x}$ and $\dddot{x}$
   We don't know anything about what these mystery operations mean—however, we have been told what the derivative of the arrow function is: If $g(x) = \vec{x}$, then the derivative is $g'(x) = \vec{x} \cdot \dddot{x}$
   Use this information about the derivative of the arrow function, along with your knowledge of derivative rules to find $\frac{dy}{dx}$ for the following function: $y = \ln\left(\overleftrightarrow{2x^4+2^x}\right)$

2. For $f(x) = 3x^2 - 2x + 1$, find $\frac{df}{dx}$ using one of the formal limit definitions of the derivative:

   a. Show how the derivative should be calculated using the formula.

   b. Graph *y*, and then draw and label the various parts of the derivative definition on that graph.

3. Draw the following graphs:
   a. Draw a picture of a graph which has a point where the graph is NOT differentiable. Label that point "A."
   b. Draw a picture of a graph which has a point along a curve where it is differentiable. Label that point "B."
   c. Make up a graph for a function $f(x)$ that has one local maximum, one local minimum, and a vertical asymptote. Now, on a separate coordinate plane, draw the graph of the derivative $f'(x)$ of that graph. Label both graphs.
   d. Make up a graph for a function $g(x)$ that has a cusp and a horizontal asymptote. Now, on a separate coordinate plane, draw the graph of the derivative $g'(x)$ of that graph. Label both graphs.
   e. Make up a graph for a function $h'(x)$ that has at least one local maximum or minimum. Now, on a separate coordinate plane, draw one possible graph of $h(x)$. Label both graphs.

4. Consider the following equations. Using **only the techniques of calculus**, and **SHOWING ALL WORK**, sketch the graph. Be sure to label:
   - All *x* and *y*-intercepts
   - All relative extrema (relative maximums and minimums)
   - All points of inflection
   - All vertical, horizontal, and slant asymptotes

   a. $f(x) = \frac{x^3}{2x^2-8}$
   b. $g(x) = xe^x$

5. Draw a section of a curve so that:
   a. The original curve is becoming more positive, but the derivative is becoming more negative on the whole interval.
   b. The original curve is becoming more negative, but the derivative is becoming more positive on the whole interval.
   c. The original curve is becoming more positive, and the derivative is becoming more positive on the whole interval.
   d. The original curve is becoming more negative, and the derivative is becoming more negative on the whole interval.